SYSTEM IDENTIFICATION AND SIMULATION OF A TRIAXIAL SHAKER SYSTEM

John J. Dougherty * and Hossny El-Sherief †

TRW Systems Integration Group
San Bernardino, California 92402
john.dougherty@trw.com, hossny.el-sherief@trw.com

Abstract

A simulation of a triaxial shaker system is being constructed to evaluate system capabilities and limitations, supplement live vibration testing, support test methodology and test procedure development, and aid in making controller software modifications. Two different types of models have been implemented in the simulation. The first is a physically-based model derived from a finite element analysis together with a model-updating system identification scheme; the second is a parametric model without direct physical significance. The advantages and disadvantages of each model for this application are considered. The methods used to derive each of the models are described. Test cases were used to evaluate the effectiveness of some of the methods. Results of the system identification process are discussed. Certain methods are found to produce models that are in good agreement with measured response data from the actual shaker system.

1. Introduction

The United States Air Force has developed a triaxial shaker system for shock and vibration testing[1]. The system consists of a table upon which the test article is mounted and eight actuators placed along three orthogonal axes (see figure 1). The shaker system is designed to produce vibrations ranging in frequency from 1 to 2500 Hz. The controller for the system is capable of multiple modes of operation, including sine wave, transient waveform, random waveform, random on random, sine on random, and mission simulation. A multiaxis vibration test facility has the potential to provide great flexibility in test design. In particular, the capability to impart both translational and rotational motion on an article is a significant advance in test technology. At the same time, active control of this multiple-input, multiple-output system represents a formidable problem.

An engineering simulation of the triaxial shaker system is being constructed in order to evaluate system capabilities and limitations, supplement live vibration testing, support test methodology and test procedure development, and aid in making controller software modifications[2]. The simulation is described in section 2. Two different types of models have been implemented in the simulation. The first is a physically-based model derived from a finite element analysis together with a model-updating system identification scheme[3,4]; the second is a parametric model without direct physical significance[5]. The advantages and disadvantages of each model for this application are considered here.

System identification schemes were employed to ensure that the models used in the simulation were predictive of actual test data. For the physically-based models, model-updating system identification schemes were used. Various approaches to model updating have been proposed. For instance, Minas and Inman systematically adjusted a state space model based on measured resonant frequencies and damping ratios (modal data) by using a pole placement method[6]. Numerous other techniques have been cited in the literature. The chief advantage of this class of approaches is that *a priori* knowledge of the system can be incorporated into the identification process.

Many approaches to updating models can be classified into one of two groups, input (or equation) error formulations and output error formulations[7,8,9]. Input error formulations produce a system of equations that may be linear in the parameters to be identified, if measured responses corresponding to each degree of freedom in the model are available and if the system equations are linear in the parameters. Output error formulations produce a system of equations that are nonlinear in the parameters to be identified, but they can easily handle cases were there are fewer measurements than degrees of freedom in the model (the usual case). A difficulty with the output error formulation is successfully converging on the true values of the parameters, since the receptance function (which appears in the formulation) can be a highly nonlinear or even discontinuous function of the parameters[3,10].

Much work has been done using input error formula-

*Staff Engineer; Member, AIAA

†Manager, Avionics and Ground Systems Technology Department

tions. Most updating methods associated with these formulations require one-to-one correspondence between the analytical degrees of freedom and the experimental degrees of freedom (the measurements). This was the case considered by Mottershead[11], Mottershead et al[12], and Friswell[13]. In cases where there are fewer measurements than analytical degrees of freedom, the one-to-one correspondence can be accomplished through reduction of the analytical system matrices or expansion of the experimental data. Methods for dynamic reduction were presented by Lammens et al[10] and by D'Amabrogio and Fregolent[14]. These methods assume that no forces are applied at unmeasured degrees of freedom. This assumption is not valid for the triaxial shaker problem; an approach that is valid for the case of the triaxial shaker was presented previously[4].

For the problem of the triaxial shaker system, a model was constructed that represented the table, the eight actuators, and the eight bearings. Nominal values for the coefficients in this model were derived from a finite element model and from manufacturer's data. Unfortunately, the model was only moderately predictive of the actual response of the system over a 300 Hz band for which test data was available. Three approaches were evaluated for the purpose of updating the model coefficients in order to obtain better agreement with the test data. Two are based on input (or equation) error formulations, while the third is based on an output error formulation. These approaches are discussed in section 3. They have been applied to two different test cases; the results are discussed. Work continues on applying these approaches to the actual system.

A second modeling approach was evaluated in an attempt to circumvent problems encountered using a physically-based model. This second approach involves fitting the system response with a parametric model of the system, in this case, an autoregressive moving average (ARMA) model.

Many methods for parametric modeling have been proposed. Several involve time-domain measurements. For instance, Hu, Chen, and Wu considered the case where a time-domain input/output pair, or just an output, is available for every degree of freedom of the system[15]. They used a modified Yule-Walker algorithm to obtain autoregressive coefficients, which were then used to produce modal parameters. Jabbari and Gibson used a lattice filter to estimate the order and coefficients of an ARMA model given time-domain input and output measurements[16 17].

If the available measurements are in the frequency domain (this is the case for the much of the available triaxial shaker system data), other approaches are available. Chen, Juang, and Lee converted frequency response data to Markov parameters by way of a finite-ordered matrix fraction, and then used the Markov parameters to construct a linear state space model by way of a eigensystem realization algorithm (which uses a singular value decomposition)[18]. Jean-Christophe obtained a transfer function from modal data[19]. Again, the literature contains many other possible approaches.

For this problem, the approach of Friedlander and Porat[20] proved useful. They proposed using a modified Yule-Walker algorithm to generate ARMA coefficients (for an assumed order) from frequency response data. The entire algorithm is quite compact computationally; it involves the least-squares solution of three linear systems and a fast Fourier transform. The technique is briefly described in section 4. The results of deriving models using this technique are also presented.

Conclusions regarding the simulation of the triaxial shaker system and the techniques used to derive the models are presented in section 5.

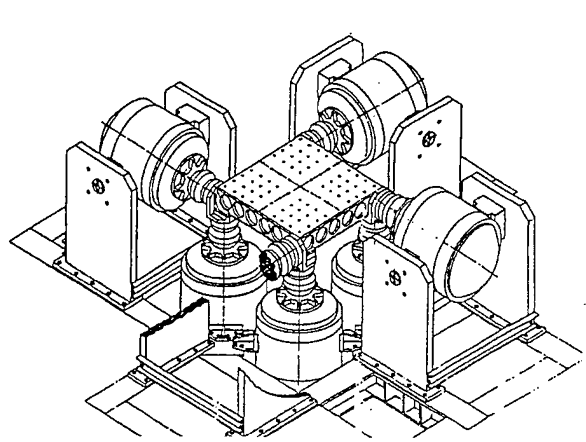

Figure 1: Triaxial Shaker Table

2. Simulation Design

An engineering simulation of the triaxial shaker system is being constructed. The system's actuators are Ling Electronics model 2106 electrodynamic shakers connected to double-spherical hydrostatic bearings, which are in turn connected to the 40-inch square magnesium table. Each shaker consists of a coil, mounted on a fixed base, and an armature.

Two different models have been implemented. The first is physically-based model consisting of a table model and models for the eight actuators, including the bearings. The table is modeled by a linear system of springs and masses derived from a finite element model. The original form of this system contained 7,392 structural nodes for a total of 44,352 degrees of freedom[21 22]. Since this results in mass and stiffness matrices with nearly two billion elements each, this system is not practical for computer simulation. Instead, the table system was reduced to 66 degrees of freedom through a generalized dynamic

reduction (an extension to Guyan reduction)[23 24]. The 66 degrees of freedom represent six degrees of freedom at each of the eight actuator attachment points (known as control points) and eighteen additional degrees of freedom used to fit the original modal data. The reduced model matches the full model to within one percent in frequency for each of the first four bending modes and to within five percent for each of the next six modes. The models for the actuators consist of a system of springs, masses, and dampers. Values for the various coefficients were estimated using manufacturer's data and the techniques described in section 3.

The table bending model, even after reduction, is large, has many high frequency modes, and is computationally expensive to integrate. Rather than simulate all of the modes, only a subset is actually simulated. First, the table coordinates are transformed into modal coordinates. Only the modal states with natural frequencies of less than some cutoff frequency are actually integrated; the others are set to zero. The cutoff frequency is chosen as five times the highest frequency of interest.

A fixed-step-size Runge-Kutta scheme was used as the baseline numerical integrator[25]. Even after the highest frequency bending modes were neglected, this method required a small step size. Experiments were conducted with other schemes, including a variable-step Runge-Kutta scheme, the Adams-Moulton method, and Gear's backward differentiation formula (BDF) method[25 26]. For low frequency inputs, the high frequency modes decay after only a fraction of a period and the system becomes numerically stiff[27 28]. The implicit methods (Adams-Moulton and Gear's BDF) tended to be more efficient after the initial transients decayed, but displayed a sensitivity to sudden changes in the external forces.

The second model has the form of an autoregressive moving average function. The coefficients were derived from available test data, as described in section 4. Since the ARMA model produces the system response directly, no numerical integration is necessary, and the simulation executes rapidly.

A simple control algorithm was implemented in the simulation. This allowed for preliminary studies, in particular comparisons to actual testing, to be conducted. Eventually, the actual control algorithms used in the triaxial shaker system will be integrated into the simulation.

3. Identification of a Physically-Based Model

An initial physically-based model of the triaxial shaker system was developed from a finite element analysis and manufacturer data. After linearization, the model assumed a system of the form

$$\dot{x} = A(\theta)x + B(\theta)u \tag{1}$$

$$y = C(\theta)x + D(\theta)u + v \tag{2}$$

where x is the state, u is the control, y is the measurement, and v is measurement noise. In particular for this work, the state vector x consists of positions and velocities, and the measurements are accelerations. The matrices A, B, C, and D depend on the vector of parameters θ that are to be updated. For this work, the matrix D is assumed to be zero.

Three algorithms for estimating θ are considered. The first uses an output error formulation. Let $G_{ik}(\omega_n)$ be the measured frequency response of the system from the kth input control to the ith output acceleration, at $N-1$ frequencies ω_n. The model frequency response from input k to output i is

$$\hat{G}_{ik}(\omega_n) = C_i(\theta)\left(j\omega_n I - A(\theta)\right)^{-1} B_k(\theta) \tag{3}$$

The cost function for the first algorithm is

$$J_1(\theta) = \sum_{n=0}^{N-1}\sum_{i=1}^{p}\sum_{k=1}^{q}\left(|\hat{G}_{ik}(\omega_n)| - |G_{ik}(\omega_n)|\right)^2 \tag{4}$$

where q is the number of inputs and p is the number of outputs. The cost function is minimized by a Nelder-Mead simplex method, which requires no gradient information[29].

The second and third algorithms use input error formulations. Let

$$S_k(\omega_n,\theta) = \left(j\omega_n I - A(\theta)\right)\frac{\partial \hat{X}}{\partial U_k}(\omega_n,\theta) - B_k(\theta) \tag{5}$$

for $1 \le k \le q$, where $\partial\hat{X}/\partial U_k$ is estimated from the measurements[4]. Then the cost function for the second algorithm is

$$J_2(\theta) = \sum_{n=0}^{N-1}\sum_{k=1}^{q}|S_k(\omega_n,\theta)|^2 \tag{6}$$

The cost function $J_2(\theta)$ is minimized with respect to θ through a Nelder-Mead simplex search.

The third algorithm uses the same cost function, $J_2(\theta)$, as the second algorithm, but minimizes it through a least-squares approach[4].

The three algorithms were applied to two different test cases to evaluate their relative performance; those results follow. Work continues in applying the most promising of these techniques to actual shaker test data.

3.1. The First Test Case

The physical system for the first test case is depicted in figure 2. Each small block represents a spring and damper in parallel. The stiffness and damping matrices, K_1 and C_1, are composed of linear combinations of the spring and damper coefficients for the blocks indicated

in the figure. The mass matrix, M_1, is a diagonal matrix. The system matrices are then

$$A = \begin{bmatrix} 0 & I \\ -M_1^{-1}K_1 & -M_1^{-1}C_1 \end{bmatrix} \tag{7}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_2 & 0 & 0 \end{bmatrix}^T \tag{8}$$

$$C = \begin{bmatrix} A_{7i} \\ A_{8i} \end{bmatrix} \tag{9}$$

where g_1 and g_2 are control gains and where A_{7i} and A_{8i} represent the seventh and eighth rows of A.

parameter	percent error versus truth			
	initial	OE/S	IE/S	IE/LS
$K_1(1,1)$	15.0	−8.7	−6.4	0.0
$K_1(1,2)$	15.0	12.8	23.1	0.0
$K_1(1,3)$	15.0	23.9	42.3	0.0
$K_1(1,4)$	15.0	10.7	53.0	0.0
$K_1(2,2)$	15.0	18.7	9.2	0.0
$K_1(2,3)$	15.0	−28.3	−2.3	0.0
$K_1(2,4)$	15.0	−36.7	2.2	0.0
$K_1(3,3)$	15.0	−17.2	3.1	0.0
$K_1(3,4)$	15.0	11.3	−3.2	0.0
$K_1(4,4)$	15.0	3.7	2.6	0.0
$C_1(1,1)$	−15.0	34.4	17.2	0.0
$C_1(1,2)$	−15.0	13.8	11.8	0.0
$C_1(1,3)$	−15.0	−29.3	−10.7	0.0
$C_1(1,4)$	−15.0	−21.2	15.2	0.0
$C_1(2,2)$	−15.0	17.0	−10.5	0.0
$C_1(2,3)$	−15.0	0.6	−2.6	0.0
$C_1(2,4)$	−15.0	−20.0	11.2	0.0
$C_1(3,3)$	−15.0	23.0	12.9	0.0
$C_1(3,4)$	−15.0	−20.7	−133.1	0.0
$C_1(4,4)$	−15.0	−1.5	7.1	0.0
g_1	66.7	−6.2	−26.7	0.0
g_2	100.0	46.0	−0.1	0.0

Table 1: Estimated Parameters

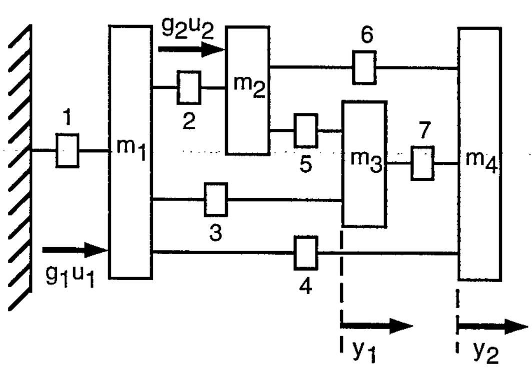

Figure 2: Test Case 1 Schematic

Results for the three algorithms for the first test case are presented in table 1 and figures 3 and 4. The parameters chosen for this test case were the elements of the symmetric matrices K_1 and C_1 and the control gains. The least squares solution to the input error formulation produces good estimates of the unknown parameters. The estimates of the parameters using the simplex solutions to both the input error and output error formulations are not as good, though the frequency response produced by the erroneous models is still close to the measured response. This suggests that the search algorithms converged on local minima away from the actual parameter values.

The least squares solution to the input error formulation (IE/LS) converged in less than ten iterations. The convergence for the simplex minimization of both the input (IE/S) and output (OE/S) formulations was somewhat slower, each requiring several thousand evaluations of the cost function (figure 4). The output error formulation converged more slowly than the input error formulation.

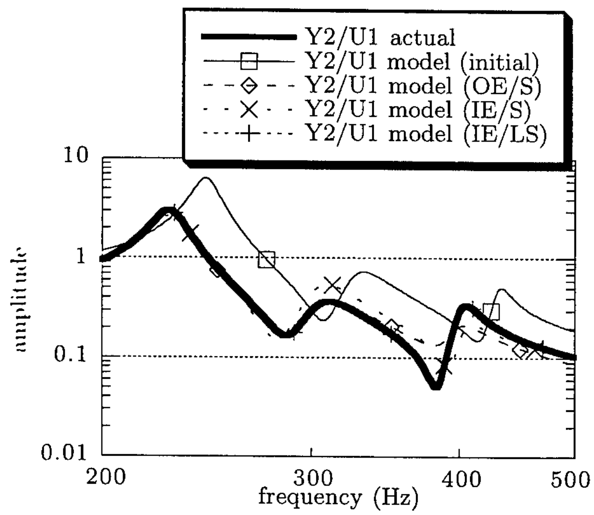

Figure 3: Test Case 1 Model Response

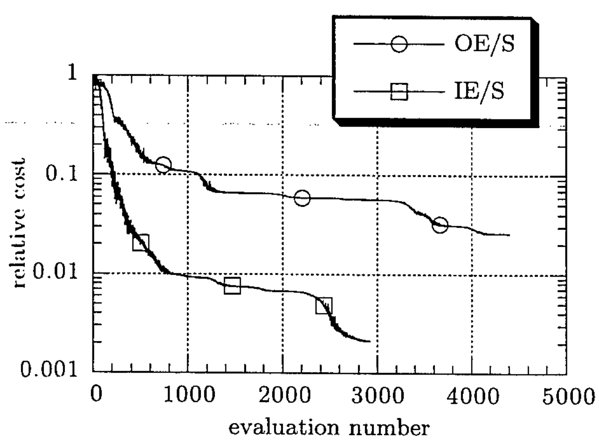

Figure 4: Test Case 1 Convergence

3.2. The Second Test Case

The second test case is a portion of a model for a triaxial shaker system. The actual system consists of eight actuators attached to a 40 inch by 40 inch square table. Four of the actuators are attached to the bottom of the table and are aligned vertically, while the other four actuators are attached to the sides and are aligned horizontally. Because the immediate objective is to evaluate the relative performance of the three identification algorithms, a simplified model of only one axis of the horizontal motion is considered. Note, however, that it is straightforward to extend the model to three dimensions[2].

The physical model for the second test case is depicted in figure 5. Each small block represents a spring and damper in parallel. Only forces and motion along the axis indicated by the arrows are considered. The springs and dampers are assumed linear in that direction. The masses represent the mass of the shaker table, while the springs and dampers represent table and actuator compliance and damping. The gain is the actuator gain, and the measurements are from accelerometers positioned as indicated. The stiffness and damping matrices, K_2 and C_2, are composed of linear combinations of the individual spring and damper coefficients, k_i and c_i, for each block indicated in the figure. The mass matrix, M_2, is a diagonal matrix.

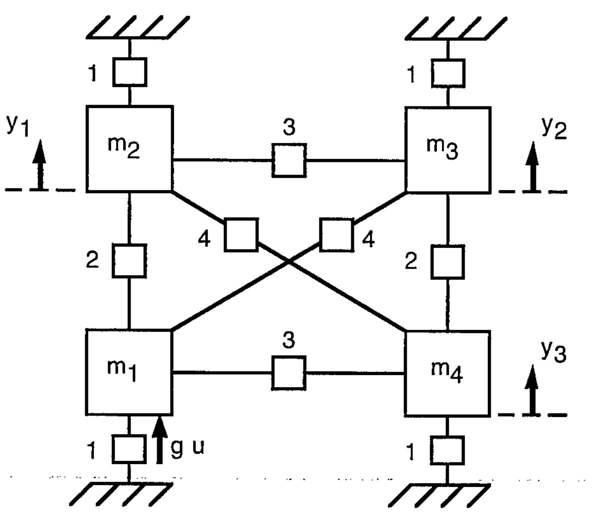

Figure 5: Test Case 2 Schematic

The system matrices are then

$$A = \begin{bmatrix} 0 & I \\ -M_2^{-1}K_2 & -M_2^{-1}C_2 \end{bmatrix} \quad (10)$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & g & 0 & 0 & 0 \end{bmatrix}^T \quad (11)$$

$$C = \begin{bmatrix} A_{6i} \\ A_{7i} \\ A_{8i} \end{bmatrix} \quad (12)$$

where g is a control gain and where A_{6i}, A_{7i}, and A_{8i} represent the sixth, seventh, and eighth rows of A.

parameter	percent error versus truth			
	initial	OE/S	IE/S	IE/LS
k_1	15.00	−0.01	0.01	0.00
k_2	15.00	−0.07	0.02	0.00
k_3	15.00	0.67	−0.17	0.00
k_4	15.00	0.02	−0.00	0.00
c_1	−15.00	−0.19	−0.03	0.00
c_2	−15.00	−0.52	0.33	0.00
c_3	−15.00	0.75	0.18	0.00
c_4	−15.00	−0.22	−0.06	0.00
g	233.33	−0.08	0.00	0.00

Table 2: Estimated Parameters

Results for the three algorithms for the second test case are presented in table 2 and figures 6 and 7. The parameters chosen for this test case were the spring coefficients, the damping coefficients, and the control gain. All three algorithms produce good estimates of the unknown parameters. The resulting frequency responses are likewise good.

The least squares solution to the input error formulation converged in less than ten iterations. The convergence for the simplex minimization of the input error formulation took hundreds of evaluations of the cost function, while the output error formulation required several thousand (figure 7).

A comparison of the computational effort required for each algorithm, for both test cases, is given in figure 8. The computational effort for the output error algorithm is driven by the need to repeatedly invert the matrix in (3), while the effort for the two input error algorithms is driven by the need to pseudoinvert another matrix. Although the simplex minimization of the input error formulation requires fewer cost function evaluations than the output error approach, the expense of the pseudoinversion results in a greater overall computational burden. Because the least-squares minimization of the input error formulation converges in so few steps, its total cost is lowest, in spite of the pseudoinversion. Note also that the dependence on a pseudoinversion makes the input error formulations expensive for system matrices with a large dimension.

Test case 2 required less computational effort than test case 1. This is due in part to the parameterization of the problems. For test case 1, no assumption was made regarding the structure of the stiffness and damping matrices, other than that of dimension. The elements themselves of these matrices were taken as the unknown parameters. For test case 2, a certain structure, including some symmetry, was assumed for the stiffness and damping matrices, and the parameters included unknown values within that structure. This not only decreased the number of unknown parameters (relative to test case 1),

but it also constrained the search for a minimizing parameter set in such a way as to encourage convergence.

The simplex method of minimizing a cost function is probably not the most efficient algorithm, but it has the advantage of simplicity in implementation. For this problem, it also appeared to have the advantage of being better able to avoid local minima, as compared to other techniques that were considered, including a Broyden-Fletcher-Goldfarb-Shanno variable metric (quasi-Newton) method[29].

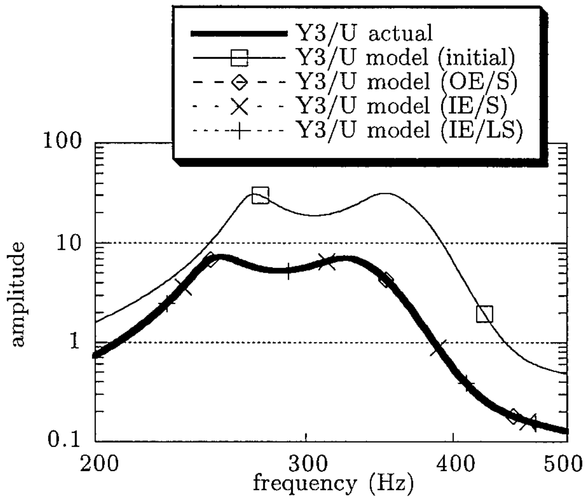

Figure 6: Test Case 2 Model Response

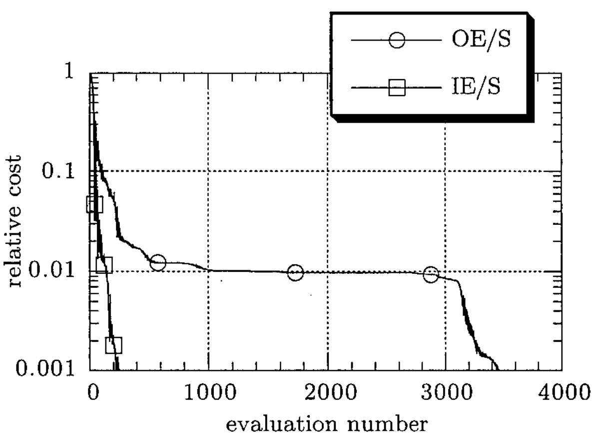

Figure 7: Test Case 2 Convergence

4. Identification of a Parametric Model

Rather than using a model based on a finite element model of the table and models for the actuators, an autoregressive moving average (ARMA) model can be used for each of the input/output pairs. This approach results in a model that compares well with the test data. The disadvantages of this approach are that the parameters themselves have no physical significance and that there is no convenient way to incorporate *a priori* knowledge (specifically, the finite element model) into the model.

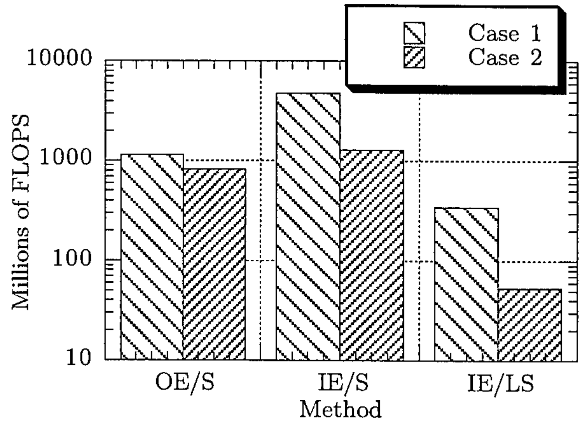

Figure 8: Computational Effort

In this approach, the identification problem is to find a model transfer function $\hat{G}(z)$, which has the form

$$\hat{G}(z) = \frac{\hat{b}(z)}{\hat{a}(z)} \tag{13}$$

$$\hat{a}(z) = \sum_{i=1}^{\hat{p}} \hat{a}_i z^{-i} \tag{14}$$

$$\hat{b}(z) = \sum_{i=1}^{\hat{p}} \hat{b}_i z^{-i} \tag{15}$$

such that the error

$$e = \hat{G}_k - G_k \tag{16}$$

is minimized, where

$$\hat{G}_k = \left|\hat{G}(z)\right|_{z=e^{j\left(\frac{2\pi}{N}\right)k}} \tag{17}$$

$$G_k = \left|G(z)\right|_{z=e^{j\left(\frac{2\pi}{N}\right)k}} \tag{18}$$

for $k = 0, 1, \ldots, N-1$. The true transfer function $G(z)$ is assumed to be of the form

$$G(z) = \frac{b(z)}{a(z)} \tag{19}$$

$$a(z) = \sum_{i=1}^{p} a_i z^{-i} \tag{20}$$

$$b(z) = \sum_{i=1}^{q} b_i z^{-i} \tag{21}$$

The procedure used is that of Friedlander[20 5]. First, an unwindowed correlation sequence is generated by taking the discrete inverse Fourier transform of the square of the desired amplitude response. Next, a windowed correlation sequence is formed by multiplying the unwindowed sequence by a Hamming window, and then the autoregressive coefficients $\hat{a}$ are computed. In order to

compute the moving average coefficients $\hat{b}$, an additive decomposition is performed, followed by the construction of the complex cepstrum of the estimated transfer function $\hat{G}(z)^{30}$, from which the moving average coefficients are derived.

Figures 9 through 12 show a comparison between test data and the ARMA model. Figures 9 and 10 compare the frequency response from test data and the model frequency response for three cases, for $\hat{p}$ equal to 5, 10, and 20, for testing done with an empty table. The model shows good agreement for $\hat{p} = 5$ and excellent agreement for higher $\hat{p}$. Results for the other fourteen input/output pairs were similarly good. (Due to symmetry, the responses of the eight control points to the eight actuators can be characterized by sixteen input/output pairs: eight responses to a horizontal actuator and eight responses to a vertical actuator.)

Figures 11 and 12 show the results for a table with a payload attached. The payload is a test fixture for an NS20 guidance system. Once again, the model shows good agreement with the test data.

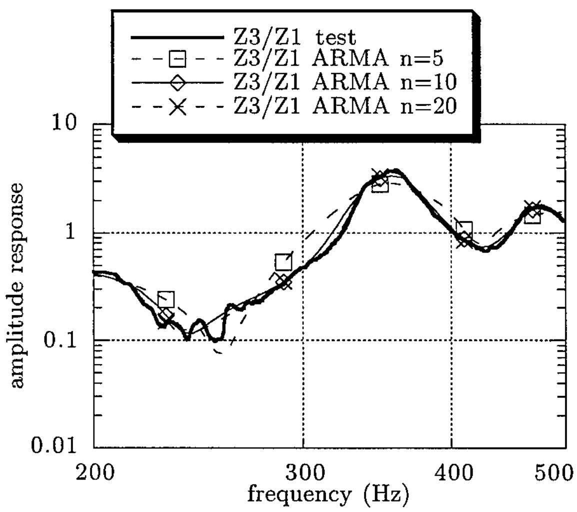

Figure 9: ARMA Estimation, Empty Table (Vertical Excitation)

5. Conclusions

An engineering simulation of the triaxial shaker system allows for the evaluation of system capabilities and limitations, the support of test methodology and test procedure development, and the testing of controller software modifications. The implementation of the system models is designed to produce rapid execution.

Two different modeling approaches were considered. For the first approach, which used a physically-based model, two different formulations were used for updating the model with respect to test data. The output error formulation is hindered by the presence of significant non-

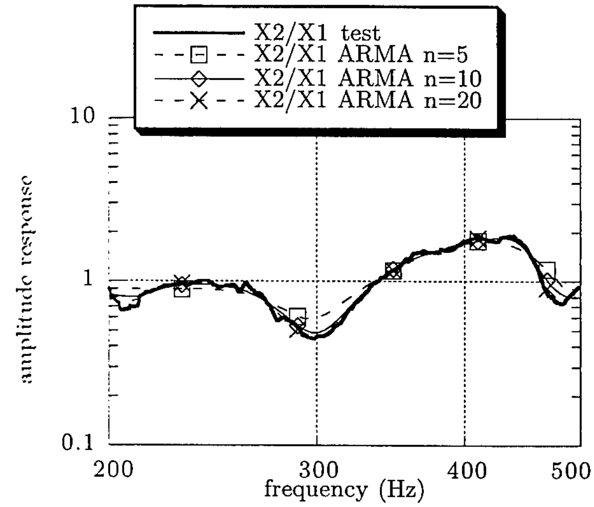

Figure 10: ARMA Estimation, Empty Table (Horizontal Excitation)

linearities in its cost function, and it displayed only moderate performance for the test cases. The input error formulations performed very well for the two test cases. A least-squares approach to minimizing the cost function produced by the input error formulation was particularly effective. Work continues on applying the most promising of these techniques to the triaxial shaker system.

The second modeling approach used a parametric model, in particular, an autoregressive moving average model. This technique produced a model that agreed quite well with the test data. The major disadvantage to the parametric approach is that, unlike the physically-based model, there is no direct physical interpretation of the resulting coefficients. This precludes using the parametric model for studies of the system behavior under perturbations of physical characteristics. Nevertheless, this approach is useful for evaluating the capabilities and limitations of the triaxial shaker system.

Acknowledgment

This work has been performed in support of Lt. Colonel Stu Flood, PMT Chief of the Sustainment Support Organization, and Mike Shafer, Survivability and Vulnerability Integration Center facility manager, under contract number F42610-92-C-0100. The authors are grateful to Dick Johnson, TRW project manager, and Del Wilson, Boeing special projects engineer, for their assistance. The table drawing and table test data were produced for the United States Air Force by Wyle Laboratories.

References

[1] Shafer, M. J., Wilson, D. R., and Johnson, R. V., "New ICBM shock and vibration test capabilities at OO-

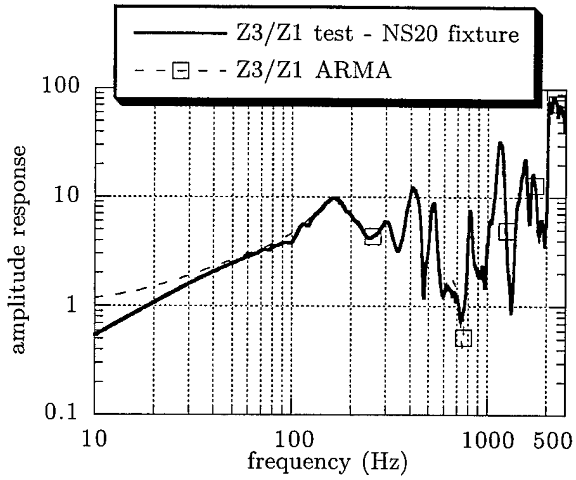

Figure 11: ARMA Estimation, Loaded Table (Vertical Excitation)

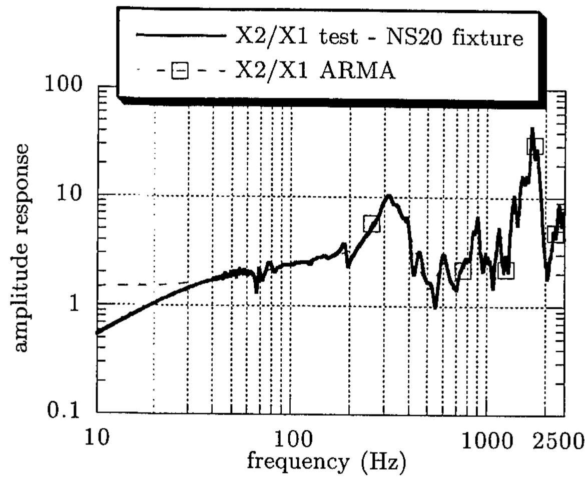

Figure 12: ARMA Estimation, Loaded Table (Horizontal Excitation)

ALC's Survivability and Vulnerability Integration Center," *Proceedings of the Joint Services Data Exchange for Guidance, Navigation, and Control*, 1994.

[2] Dougherty, J. J., El-Sherief, H., and Nguyen, T., "Rigid and flexible body modeling for a triaxial shaker control system," *Proceedings of the ASME Design Engineering Technical Conference*, American Society of Mechanical Engineers, New York, pp. 369–374, 1995.

[3] Dougherty, J. J. and El-Sherief, H., "Modeling and identification of a triaxial shaker control system," *Proceedings of the Conference on Control Applications*, The Institute of Electrical and Electronics Engineers, Piscataway, NJ, pp. 884–889, 1995.

[4] Dougherty, J. J. and El-Sherief, H., "Parameter Identification for a Triaxial Shaker System," *Proceedings of the Conference on Control Applications*, The Institute of Electrical and Electronics Engineers, Piscataway, NJ, pp. 558–563, 1996.

[5] Dougherty, J. J. and El-Sherief, H., "An ARMA approach for modeling a triaxial shaker system," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, American Institute of Aeronautics and Astronautics, Washington, D. C., paper AIAA-96-3718, July 1996.

[6] Minas, C. and Inman, D. J., "Model improvement by pole placement methods," *Proceedings of the ASME Conference on Mechanical Vibration and Noise*, American Society of Mechanical Engineers, New York, pp. 179–185, 1989.

[7] Fritzen, C.-P., "Identification of mass, damping, and stiffness matrices of mechanical systems," *Transactions of the American Society of Mechanical Engineers, Journal of Vibration, Acoustics, Stress, and Reliability in Design*, Vol. 108, pp. 9–16, 1986.

[8] Fritzen, C.-P. and Zhu, S., "Updating of finite element models by means of measured information," *Computers and Structures*, Vol. 40, No. 2, pp. 475–486, 1991.

[9] Foster, C. D. and Mottershead, J. E., "A method for improving finite element models by using experimental data: application and implications for vibration monitoring," *International Journal of Mechanical Sciences*, Vol. 32, No. 3, pp. 191–203, 1990.

[10] Lammens, S., Brughmans, M., Leuridan, J., Heylen, W., and Sas, P., "Updating of dynamic finite element models based on experimental receptances and the reduced analytical dynamic stiffness matrix," *Proceedings of the ASME Conference on Mechanical Vibration and Noise*, American Society of Mechanical Engineers, New York, 1995, pp. 1191–1200.

[11] Mottershead, J. E., "On the optimal correction of structural dynamic models," *Proceedings of the ASME Conference on Mechanical Vibration and Noise*, American Society of Mechanical Engineers, New York, pp. 107–112, 1991.

[12] Mottershead, J. E., Tee, T. K., Foster, C. D., and Stanway, R., "An experiment to identify the structural dynamics of a portal frame," *Proceedings of the ASME Conference on Mechanical Vibration and Noise*, American Society of Mechanical Engineers, New York, pp. 109–115, 1989.

[13] Friswell, M. I., "Updating physical parameters from frequency response function data," *Proceedings of the ASME Conference on Mechanical Vibration and Noise*, American Society of Mechanical Engineers, New York, pp. 393–400, 1989.

[14] D'Ambrogio, W. and Fregolent, A., "A comparison between the use of physical parameters and correction factors in dynamic model updating," *Proceedings of the ASME Conference on Mechanical Vibration and*

Noise, American Society of Mechanical Engineers, New York, 1995, pp. 1181–1190.

[15] Hu, S., Chen, Y. B., and Wu, S. M., "Multi-output modal parameter identification by vector time series modeling," *Proceedings of the ASME Conference on Mechanical Vibration and Noise*, American Society of Mechanical Engineers, New York, pp. 259–265, 1989.

[16] Jabbari, F. and Gibson, J. S., "Vector-channel lattice filters and identification of flexible structures," *IEEE Transactions on Automatic Control*, Vol. 33, No. 5, pp. 448–456, 1988.

[17] Jabbari, F. and Gibson, J. S., "Adaptive identification of a flexible structure by lattice filters," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 4, pp. 548–554, 1989.

[18] Chen, C., Juan, J., and Lee, G., "Frequency domain state-space system identification," *Journal of Vibration and Acoustics*, Vol. 116, pp. 523–528, 1994.

[19] Jean-Christophe, A., "How to measure modal and effective parameters by performing an environmental test," *Proceedings of the Institute of Environmental Sciences*, pp. 225–234, 1987.

[20] Friedlander, B. and Porat, B., "The modified Yule-Walker method of ARMA spectral estimation," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 20, No. 2, pp. 158–172, 1984.

[21] Holms, B. A., "Minuteman three-axis shaker table reduced matrices," technical report F343.BAH.94-008, TRW Systems Integration Group, San Bernardino, CA, 1994.

[22] Araiza, R. J., "Three-axes shaker table dynamic model," technical report F343.RJA.94-002, TRW Systems Integration Group, San Bernardino, CA, 1994.

[23] Gockel, M. A., editor, *MSC/NASTRAN handbook for dynamic analysis*, MacNeal-Schwendler Corp., Los Angeles, 1983.

[24] Guyan, R. J., "Reduction of stiffness and mass matrices," *AIAA Journal*, Vol. 3, No. 2, 1965.

[25] Dahlquist, G. and Björck, Å., *Numerical Methods*, Prentice-Hall, Englewood Cliffs, NJ, pp. 342–355, 1974.

[26] Gear, C. W., *Numerical Initial Value Problems in Ordinary Differential Equations*, Prentice-Hall, Englewood Cliffs, NJ, 1971.

[27] Shampine, L. F. and Gear, C. W., "A user's view of solving stiff ordinary differential equations," *SIAM Review*, Vol. 21, No. 1, pp. 1–17, 1979.

[28] Hundsdorfer, W. H., *The Numerical Solution of Nonlinear Stiff Initial Value Problems: An Analysis of One Step Methods*, Centrum voor Wiskunde en Informatica, Amsterdam, The Netherlands, 1980.

[29] Press, W. H., Flannery, B. P., Teukolsky, S. A., and Vetterling, W. T., *Numerical Recipes–The Art of Scientific Computing*, Cambridge University Press, New York, NY, 1990.

[30] Oppenheim, A. V. and Schafer, R. W., *Digital Signal Processing*, Prentice-Hall, Englewood Cliffs, NJ, Ch. 10, 1975.

[31] El-Sherief, H., Thai, K., and Whitmer, G., "Design, simulation and analysis of a large scale hydraulic stabilization control system," *Proceedings of the American Control Conference*, Institute of Electrical and Electronic Engineers, Piscataway, NJ, pp. 2219–2223, 1993.

[32] Stroud, R. C. and Hamma, G. A., "Multiexciter and multiaxis vibration exciter control systems," *Sound and Vibration*, Vol. 22, No. 4, pp.18–28, 1988.

[33] Fitz-Coy, N. and Chatterjee, A., "Actuator placement in multi-degree-of-freedom vibration simulators," *Shock and Vibration*, Vol. 1, No. 3, pp. 279–287, 1994.

[34] Smallwood, D. O., "A random vibration control system for testing a single test item with multiple inputs," *Society of Automotive Engineers Transactions*, pp. 4571–4577, 1982.

[35] Smallwood, D. O., Woodall, T. D., and Buksa, E. J., "Minimum drive requirements for a multiple input multiple output linear system," *Proceedings of the Institute of Environmental Sciences*, pp. 295–301, 1986.

[36] Paez, T. L., Smallwood, D. O., and Buksa, E. J., "Random control at n^2 points using n shakers," *Proceedings of the Institute of Environmental Sciences*, pp. 271–275, 1987.

[37] Anderson, M. C., "Triaxial shock and vibration system Wyle bearing: evaluation test results," technical report WTM-81620.2, Wyle Laboratories, El Segundo, CA, 1992.

PLEASE CHECK THE APPROPRIATE BLOCK BELOW: M97-03-2010

☐ ________ copies are being forwarded. Indicate whether Statement A, B, C, D, E, F, or X applies.

☒ DISTRIBUTION STATEMENT A:
APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED

☐ DISTRIBUTION STATEMENT B:
DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES ONLY; (Indicate Reason and Date). OTHER REQUESTS FOR THIS DOCUMENT SHALL BE REFERRED TO (Indicate Controlling DoD Office).

☐ DISTRIBUTION STATEMENT C:
DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES AND THEIR CONTRACTORS; (Indicate Reason and Date). OTHER REQUESTS FOR THIS DOCUMENT SHALL BE REFERRED TO (Indicate Controlling DoD Office).

☐ DISTRIBUTION STATEMENT D:
DISTRIBUTION AUTHORIZED TO DOD AND U.S. DOD CONTRACTORS ONLY; (Indicate Reason and Date). OTHER REQUESTS SHALL BE REFERRED TO (Indicate Controlling DoD Office).

☐ DISTRIBUTION STATEMENT E:
DISTRIBUTION AUTHORIZED TO DOD COMPONENTS ONLY; (Indicate Reason and Date). OTHER REQUESTS SHALL BE REFERRED TO (Indicate Controlling DoD Office).

☐ DISTRIBUTION STATEMENT F:
FURTHER DISSEMINATION ONLY AS DIRECTED BY (Indicate Controlling DoD Office and Date) or HIGHER DOD AUTHORITY.

☐ DISTRIBUTION STATEMENT X:
DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES AND PRIVATE INDIVIDUALS OR ENTERPRISES ELIGIBLE TO OBTAIN EXPORT-CONTROLLED TECHNICAL DATA IN ACCORDANCE WITH DOD DIRECTIVE 5230.25, WITHHOLDING OF UNCLASSIFIED TECHNICAL DATA FROM PUBLIC DISCLOSURE, 6 Nov 1984 (Indicate date of determination). CONTROLLING DOD OFFICE IS (Indicate Controlling DoD Office).

☐ This document was previously forwarded to DTIC on ____________ (date) and the AD number is ________________.

☐ In accordance with the provisions of DoD instructions, the document requested is not supplied because:

☐ It is TOP SECRET.

☐ It is excepted in accordance with DoD instructions pertaining to communications and electronic intelligence.

☐ It is a registered publication.

☐ It is a contract or grant proposal, or an order.

☐ It will be published at a later date. (Enter approximate date, if known.)

☐ Other. (Give Reason.)

John J. Dougherty
Print or Typed Name

John J Dougherty 4 Dec 96
Authorized Signature Date

909 - 382 - 6774
Telephone Number

System Identification and Simulation of a Triaxial Shaker System #48

7A